Lecture 8

Part 1 P-N Junction
A. Junction at equilibrium
a. Junction Formation
b. Big three plots (ρ, E, V)
c. Band diagram
d. Depletion Calculation

Lecture 9

Part 1 P-N Junction
B. Junction at non-equilibrium
a. Forward & Reverse Bias
b. Carrier Distribution
c. Current Calculation
d. Capacitance in Junction

Lecture 10

Part 2 M-S Junction
a. Schottky Contact
b. Ohmic Contact
c. Schottky Barrier Height & Contact Resistance

Lecture 11~14

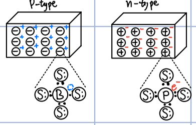
Part 3 Applications
a. Fabrication
b. Solar Cells, Photodiodes LED, and Lasers
c. Zener Diode (Device Breakdown)

Then Midterm 10/28

Part1 Review

a. Junction Formation

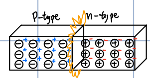
Before Contact



majority: p_p (h^+ in p-type) majority: n_n (e^- in n-type)

minority: n_p (e^- in p-type) minority: p_n (h^+ in n-type)

After Contact



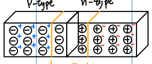
h^+ in p-type diffuses to n-type e^- in n-type diffuses to p-type

⇒ Recombination at the interface

but after free carrier leave, dopants ① become ionized ② remain fixed in lattice...

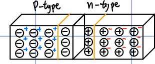
⇒ Create a Region that has no free carriers!

ie. After Contact



Depletion Region Formation

And because of these charges...



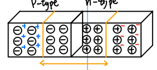
E-field exists!

So that means more e^-, h^+ diffuse to the other side ($J_n = q \cdot D_n \frac{dn}{dx}$) direction ($\because e^-$ movement ←)

⇒ more fixed charges in this region (Expanding)

⇒ Stronger E-field ⇒ Larger Drift current ($J_n = q \cdot n \cdot \mu \cdot E$) direction

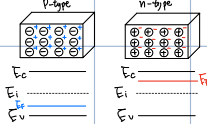
So: Depletion Region stops expanding when ⟶ $J_{diffusion} = J_{drift}$ (Equilibrium)



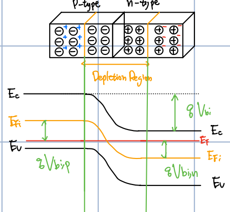
Depletion Region

c. Band Diagram

Before Contact



After Contact



Math:

$\because$ no net current can flow across the junction at equilibrium

$J_p(\text{drift}) + J_p(\text{diffuse}) = 0$

$J_n(\text{drift}) + J_n(\text{diffuse}) = 0$

ie. drift and diffusion components of the hole current just cancel at equilibrium

$J_p(x) = q\mu_p p(x)E(x) - qD_p\frac{dp(x)}{dx} = 0$

$\rightarrow \frac{\mu_p}{D_p}E(x) = \frac{1}{p(x)}\frac{dp(x)}{dx} \xrightarrow[E(x) = -\frac{dV(x)}{dx}]{} -\frac{q}{kT}\frac{dV(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx} \xrightarrow{\text{since we are interested in } V_p \text{ to } V_n} -\frac{q}{kT}\int_{V_p}^{V_n} dV = \int_{p_p}^{p_n}\frac{1}{p}dp \longrightarrow -\frac{q}{kT}(V_n - V_p) = \ln p_n - \ln p_p = \ln\frac{p_n}{p_p}$

and Einstein Relationship

$\longrightarrow V_{bi} = \frac{kT}{q}\ln\frac{p_p}{p_n}$

or u can use

$qV_{bi,n} = E_F - E_i = kT\ln\frac{n_n}{n_i}$ ... we used in class

$qV_{bi,p} = E_i - E_F = kT\ln\frac{p_p}{n_i}$

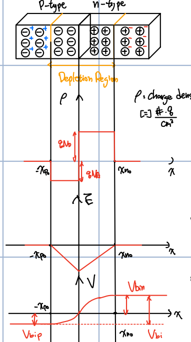
$qV_{bi} = qV_{bi,n} + qV_{bi,p}$

$\because V_{bi} = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{N_a}{n_i^2/N_D} = \boxed{\frac{kT}{q}\ln\frac{N_a N_D}{n_i^2}}$

also, we know that $\frac{p_p}{p_n} = e^{\frac{qV_0}{kT}}$ $\xrightarrow{\because p_p n_p = n_i^2 = p_n n_n}$ $\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{\frac{qV_0}{kT}}$ *

if we know V ∋ we can find the minority concentration!

b. Big three Plots : In order to understand how wide the DR is
\+
d. Depletion Region



ρ: charge density [=] $\frac{\#\cdot q}{cm^3}$ ⇒ Charge neutrality

$-Q_{in\ p} = +Q_{in\ n}$

$|-q \cdot N_A \cdot A \cdot x_p| = qN_D \cdot A \cdot x_n$ ⇒ $\boxed{\frac{x_n}{x_p} = \frac{N_A}{N_D}}$

Poisson:

$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$ ⇒ $E(x) = \frac{qN_D}{\varepsilon}(x - x_n)$, $E(x) = -\frac{qN_A}{\varepsilon}(x - x_p)$ ⇒ $E_{max} = -\frac{qN_D x_{n0}}{\varepsilon} = -\frac{qN_A x_{p0}}{\varepsilon}$

Poisson:

$E = -\frac{dV}{dx}$ → So V_{bi} = integral [E area]

$V_{bi} = V_{bi,p} + V_{bi,n}$ = left triangle + right triangle

$= \frac{1}{2}\cdot\text{base}\cdot\text{height}$ ⇒ slope $= \frac{\text{height}}{\text{base}}$ ⇒ height = base · slope

$= \frac{1}{2}\cdot x_n \cdot x_n \cdot \frac{qN_D}{\varepsilon} + \frac{1}{2}x_p \cdot x_p \cdot \frac{qN_A}{\varepsilon}$

$= \frac{1}{2}x_n^2\frac{qN_D}{\varepsilon} + \frac{1}{2}x_p^2\frac{qN_A}{\varepsilon} = \frac{1}{2}\frac{q}{\varepsilon}(x_p^2 \cdot N_A + x_n^2 N_D)$

$\because x_n = \frac{N_A}{N_A+N_D}W$ $x_p = \frac{N_D}{N_A+N_D}W$

$W = \frac{N_A}{N_D}x_p + x_p$

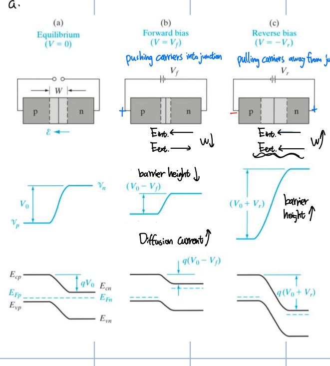
$W = \frac{N_A+N_D}{N_D}x_p$

$x_p = \frac{N_D}{N_A+N_D}W$

$= \frac{q}{2\varepsilon}\frac{N_A N_D}{(N_A+N_D)^2}W^2(N_A+N_D)$ ⇒ $\boxed{W = \sqrt{\frac{2\varepsilon}{q}V_{bi}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$

Part1 Review

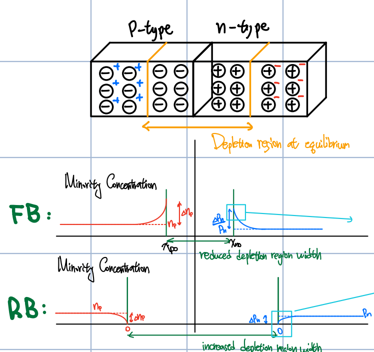
a.



$\boxed{W = \sqrt{\frac{2\varepsilon}{q}(V_{bi} - V_a)\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$

V_a: biasing voltage

b.



Why higher minority concentration at the junction?

$\because$ low-level injection from the other side

then decay exponentially with distances

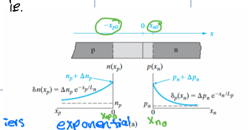
($\Delta n = \Delta n_0 \exp(\frac{-x}{L_n})$)

Why "0" zero minority concentration at the junction?

math POV: $\Delta P_n = P_n[\exp(\frac{qV_a}{kT}) - 1]$, $\because V_a = -V_r$, $\Delta P_n = -P_n$

physic POV: carriers at the edge get swept to the other side due to large E-field

ie.



Distribution of excess minority carriers that have been steady-state injected in FB

$\delta n(x_p) = \Delta n_p e^{-x_p/L_n} = n_p(e^{qV/kT} - 1)e^{-x_p/L_n}$

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$

c. Current Calculation

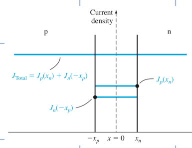
Ideal pn Junction Current Derivation

basis of assumption!

① Total current is Constant throughout the entire pn structure

② The individual e^- and h^+ current are continuous functions through the pn structure

③ In depletion region, individual e^- and h^+ currents are constant



the total current in junction

= Sum(individual e^- and h^+ current) constant.

i.e. Total pn junction current = h^+ at $x=x_n$ + e^- at $x=-x_p$ "diffusion current"

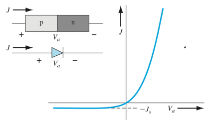
$$\therefore J_p(x_n) = -qD_p \frac{dp_n(x)}{dx}\Big|_{x=x_n} = -qD_p \frac{d\delta p_n(x)}{dx}\Big|_{x=x_n} = \frac{qD_p p_{no}}{L_p}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

$$J_n(-x_p) = qD_n \frac{dn_p(x)}{dx}\Big|_{x=-x_p} = qD_n \frac{d\delta n_p(x)}{dx}\Big|_{x=-x_p} = \frac{qD_n n_{po}}{L_n}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

And $J_{tot} = J_n(-x_p) + J_p(x_n)$

$$= \left[\frac{qD_n n_{po}}{L_n} + \frac{qD_p p_{no}}{L_p}\right]\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

$$= J_{saturation}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$



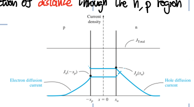
$$\therefore I_{diode} = A\cdot q\left[\frac{D_n n_{po}}{L_n} + \frac{qD_p p_{no}}{L_p}\right]\left[\exp\left(\frac{qV_a}{kT}\right)-1\right] \#$$

Summary of physics

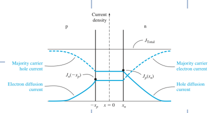
if we determine the minority carrier diffusion current as a function of distance through the n, p region

$$J_p(x) = \frac{qD_p p_{no}}{L_p}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]\exp\left(\frac{x_n - x}{L_p}\right),\ (x \geq x_n)$$

$$J_n(x) = \frac{qD_n n_{po}}{L_n}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]\exp\left(\frac{x_p + x}{L_n}\right),\ (x \leq -x_p)$$

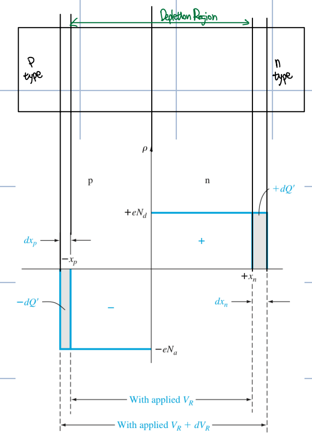


But! total current through the pn Junction is constant ⇒ majority carrier drift current



d. Junction (Depletion) Capacitance

Under Reverse Bias



Under Reverse Bias : when $V_R \longrightarrow V_R + dV_R \longrightarrow$ Depletion region expands dx
($\because V_R < 0$)

Junction Capacitance :

$$C = \frac{dQ}{dV_R} \Rightarrow dQ = q\cdot N_D \cdot dx_n = q\cdot N_A \cdot dx_p \longrightarrow \text{extra charges bc of } dV_R$$

$$C_j = \frac{dQ_n}{dV_R} = \frac{dQ_p}{dV_R} = \left|\frac{dQ_n}{dx_n}\frac{dx_n}{dV_R}\right| = \left|qN_D\frac{dx_n}{dV_R}\right|.$$

$$x_n = \frac{N_A}{N_A + N_D}\sqrt{\frac{2\varepsilon}{q}(V_{bi}-V_R)\left(\frac{1}{N_A}+\frac{1}{N_D}\right)} \Rightarrow \frac{dx_n}{dV_R} = -\sqrt{\frac{\varepsilon}{2q}\frac{1}{(V_{bi}-V_R)}\frac{N_A}{N_D(N_A+N_D)}}$$

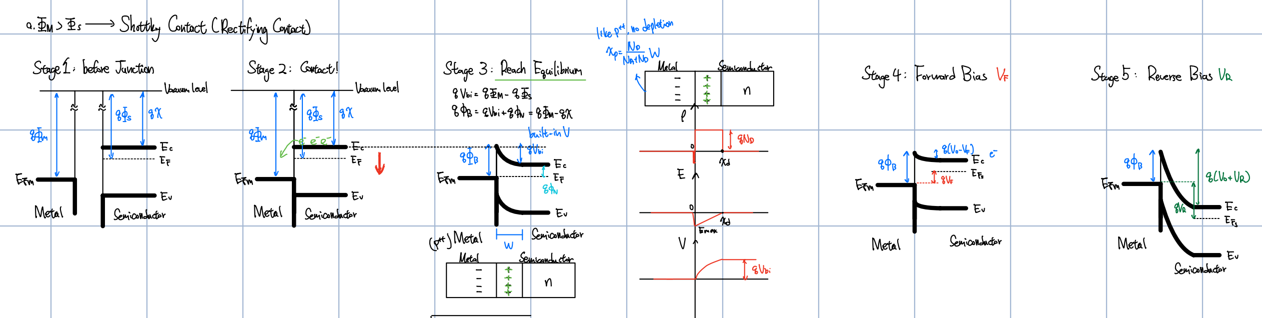
$$\therefore C_{j_{FB}} = \sqrt{\frac{q\varepsilon}{2(V_{bi}-V_R)}\frac{N_D N_A}{N_A + N_D}} = \frac{\varepsilon}{W} \text{ per unit area}$$

Part2 Review

Summary of p-type, n-type, Schottky, Ohmic, Band Diagram

① Metal - n-type Semiconductor

a. $\Phi_M > \Phi_s \longrightarrow$ Schottky Contact (Rectifying Contact)

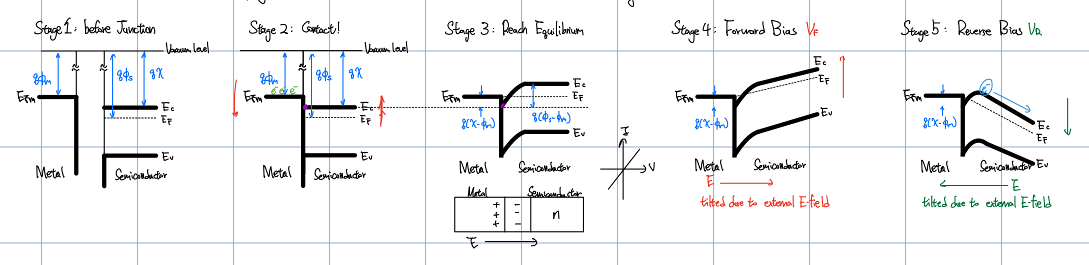


$$W = \sqrt{\frac{2\varepsilon}{q}V_{bi}\left(\cancel{\frac{1}{N_A}} + \frac{1}{N_D}\right)} = \sqrt{\frac{2\varepsilon}{q}\frac{V_{bi}}{N_D}} = x_d$$

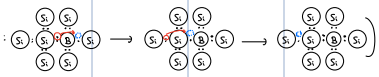
N_A large

semiconductor side only

a. $\Phi_M < \Phi_s \longrightarrow$ Ohmic Contact (Non-rectifying Contact)



② Metal - p-type Semiconductor

(Holes movement: )

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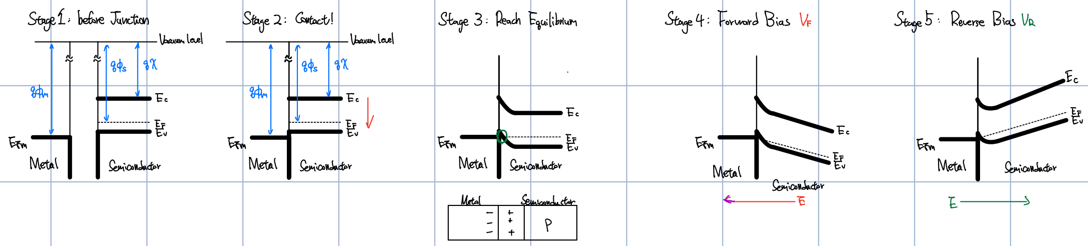


a. $\Phi_M < \Phi_S \longrightarrow$ Schottky Contact (Rectifying Contact)

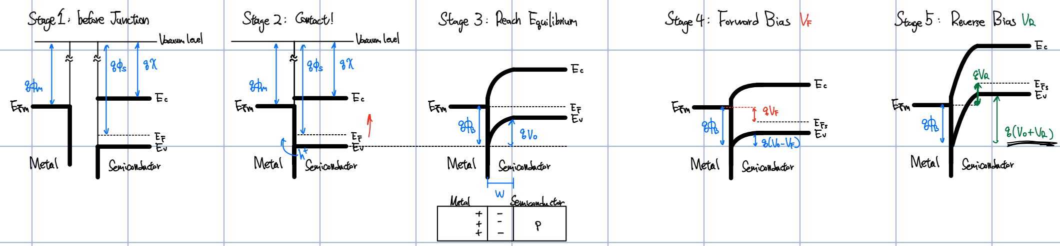
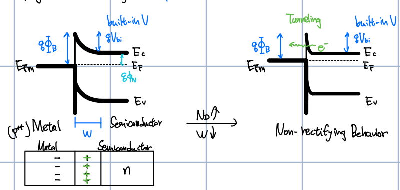


Table:

	$\Phi_M > \Phi_S$	$\Phi_M < \Phi_S$
M- n type	Schottky	Ohmic
M- p type	Ohmic	Schottky

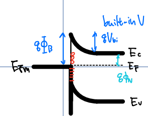
2. Special Cases

① Doping to make Schottky $\longrightarrow$ Ohmic



$W = \sqrt{\frac{2\varepsilon}{q}\frac{V_{bi}}{N_D}}$

② Fermi level Pinning



when reaching equilibrium,

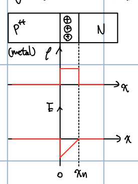
e^- from ~~semiconductor~~ surface states $\longrightarrow$ metal

Doping in semiconductor doesn't work!

$\Rightarrow$ lost DOF, fermi level is always fixed

Schottky Diode current derivation (just for your reference)

Just like an one-sided abrupt junction

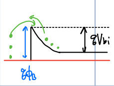


$\frac{dE}{dx} = \frac{\rho}{\varepsilon} \Rightarrow E(x) = \frac{qN_D}{\varepsilon}(x - x_n)$, $E_{max} = -\frac{qN_D x_n}{\varepsilon_s}$

$V_{bi} = \frac{1}{2}\cdot x_n \cdot \frac{qN_D x_n}{\varepsilon_s} \Rightarrow x_n = \sqrt{\frac{2\varepsilon_s}{q}V_{bi}\cdot\frac{1}{N_D}}$

Recall.. If we want to know the current, we need to know the concentration first e.g. $n_s(x)$ surface concentration

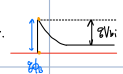
At thermal equilibrium



you definitely have some (little) higher-energy e^- can overcome the barrier

but the amount has to be the same, ie. net current = 0

$n_s = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$ ⋯ from Band Diagram ($n_s < N_c$: $E_F - E_c$)

which E_F and E_c? $\Rightarrow$ surface concentration ∴ 

ie. $E_c - E_F \simeq q\phi_{B0}$

$\therefore n_s = N_c \exp\left(-\frac{q\phi_{B0}}{kT}\right) = N_c \exp\left(-\frac{\phi_{B0}}{V_T}\right)$

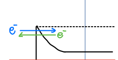
Also, $\phi_{B0} = V_{bi} + \phi_n \Rightarrow n_s = N_c \exp\left[\frac{-(V_{bi}+\phi_n)}{V_T}\right] = N_c \exp\left(-\frac{V_{bi}}{V_T}\right)\exp\left(-\frac{\phi_n}{V_T}\right) = N_D \exp\left(\frac{-V_{bi}}{V_T}\right)$

$N_D = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$

$= N_c \exp\left(-\frac{\phi_n}{V_T}\right)$

Now, we know the surface concentration, and we also know that there'll be current due to thermionic emission, just $J_{left} = J_{right}$

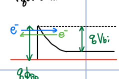
ie. $J_{m\to s} = J_{s\to m}$



since $n_s \propto J$ (current density)

$J_{m\to s} = \underbrace{C_1}_{\text{some constant}} \cdot n_s = J_{s\to m}$

Equilibrium



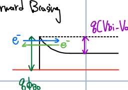
the barrier e^- sees

$J_{s\to m} = C_1 N_D \exp\left(\frac{-V_{bi}}{V_T}\right)$

$J_{m\to s} = C_1 N_c \exp\left(\frac{-\phi_{B0}}{V_T}\right)$

the barrier e^- sees

Forward Biasing



the barrier e^- sees

$J_{s\to m} = C_1 N_D \exp\left[\frac{-q(V_{bi}-V_a)}{kT}\right]$

$J_{m\to s} = C_1 N_c \exp\left(\frac{-\phi_{B0}}{V_T}\right)$

the barrier e^- sees

So,

$$J_{s\to m} = C_1 N_D \exp\left[-\frac{q(V_{bi}-V_a)}{kT}\right]$$

$$J_{m\to s} = C_1 N_c \exp\left(\frac{-\phi_{B0}}{V_T}\right)$$

Combine

$$\Longrightarrow J_D = J_{s\to m} - J_{m\to s} = C_1 N_D \exp\left[-\frac{q(V_{bi}-V_a)}{kT}\right] - C_1 N_c \exp\left[-\frac{q\phi_{B0}}{kT}\right]$$

$$= C_1 N_D \exp\left(-\frac{V_{bi}}{V_T}\right)\exp\left(\frac{V_a}{V_T}\right) - C_1 N_c \exp\left(-\frac{\phi_{B0}}{V_T}\right)$$

$$= \underline{C_1 N_D \exp\left(-\frac{\phi_{B0}}{V_T}\right)}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]$$

$$\therefore \boxed{J_D = J_{ST}\left(e^{\frac{V_a}{V_T}} - 1\right)}, \text{ where } J_{ST} = C_1 N_D \exp\left(\frac{-\phi_{B0}}{V_T}\right) = A^* T^2 \exp\left(-\frac{q\phi_{B0}}{kT}\right),\ A^* = \text{Richardson Const.} = \frac{4\pi q m_n^* k^2}{h^3}$$

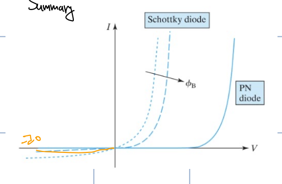
$$I_{SD} = A^* T^2 \exp\left(-\frac{q\phi_{B0}}{kT}\right)\left[\exp\left(\frac{qV_a}{kT}\right)-1\right] = I_{ST}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

I_{ST} : Saturation current for Schottky Diode

faster response : majority-carrier-device (thermionic)

More power consumption

Summary



$$I_{PN} = A \cdot q\left[\frac{D_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}\right]\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

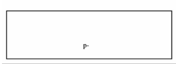
$$= I_0\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

I_0 : Saturation current for PN Diode

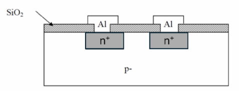
Part3 Review

a. Fabrication

Plain p-Si sample



Certain structure/device



Step1: Thermal Process

Step2: Lithography

Step3: Etching

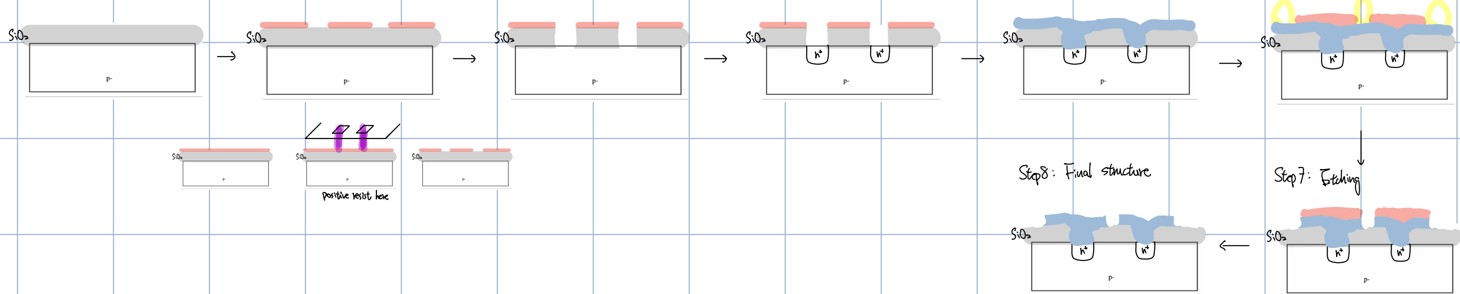
Step4: Ion Implantation

Step5: Thin-film Deposition

Step6: Lithography

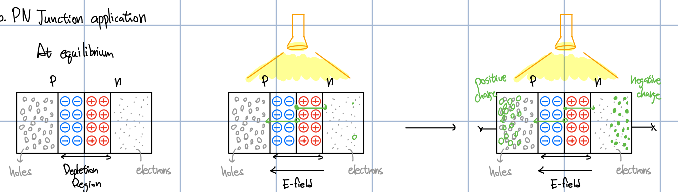
Step7: Etching

Step8: Final structure

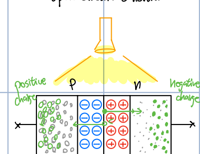


b. PN Junction application

At equilibrium



Open circuit condition

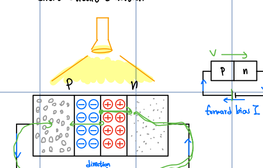


max. voltage:

V_{oc} (depends on device)

$I = 0$

Short circuit condition



max. current:

I_{sc} (negative direction, depends on light intensity)

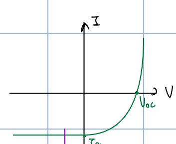
$V = 0$

math:

$$I = I_{th}\left(e^{qV/kT} - 1\right) - I_{op}$$

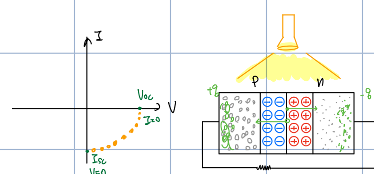
Short circuit : $V=0 \longrightarrow I = I_{th}\left(e^{qV/kT} - 1\right) - I_{op} = -I_{op}$

Open circuit : $I=0 \longrightarrow 0 = I_{th}\left(e^{qV/kT} - 1\right) - I_{op} \Rightarrow V_{oc} = \frac{kT}{q}\ln\left(\frac{I_{op}}{I_{th}}+1\right)$

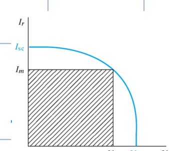


$I_{op} = qAg_{op}(L_p + L_n + W)$

Intensity ↑ ⇒ g_{op} ↑ ⇒ ⇓ shift ↑

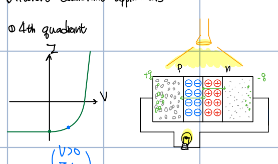


between open- and short-circuit: resistance



Different Quadrant applications

① 4th quadrant

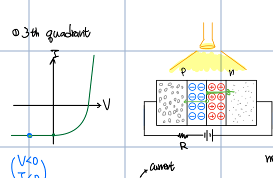


→ "solar cell"

power is delivered from junction → external circuit

$P_{max} = I_m V_m = (ff) \times I_{sc} \times V_{oc}$, $ff = \frac{I_m V_m}{I_{sc} V_{oc}}$

② 3th quadrant



→ "photodetector"

power is delivered from external circuit → junction

need to get the current related to the photons

$$\eta = \left(\frac{I_{ph}}{q}\right) / \left(\frac{P_{op}}{h\nu}\right)$$

current

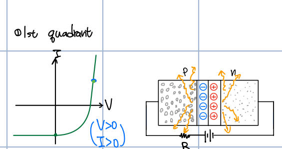
incident optical power density

external quantum efficiency

how many carriers are collected from the photons hitting on the detector

$$R = \frac{I_{ph}}{P_{op}} \Rightarrow \eta = \frac{Rhc}{q \cdot \lambda}$$

③ 1st quadrant



→ "LED, Laser"

power is delivered from external circuit → junction

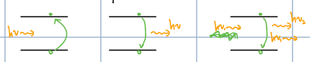
Carriers recombine near junction → emitting light

$$a_{A(1-x)B_x} = (1-x)a_A + x a_B$$

Laser

3 ways e^- interacts with photons

① absorption ② spontaneous emissio ③ Stimulated emission



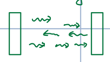
- Same wavelength
- Same phase and direction

Probability :

a. $\dfrac{\text{stimulated emission rate}}{\text{spontaneous emission}} \propto h\nu_1$

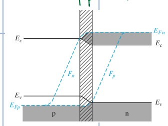
b. $\dfrac{\text{Stimulated emission rate}}{\text{Absorption rate}} \propto \dfrac{n_2}{n_1}$

↓ need cavity



Resonance : more $h\nu_1$ generated

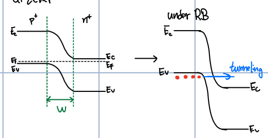
↓ need population inversion



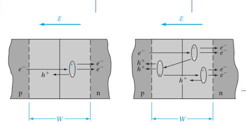
$F_n - F_p > E_g$

Breakdown

a. Zener

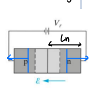


b. Avalanche



large RB → high E-field → high kinetic energy carriers
⇒ collision with lattice → more EHP

c. Punch-through

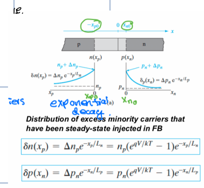


W (depletion region) fills the entire device

$$W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0 - V)}$$

Short (Narrow) Diode

ie.



Diffusion length:

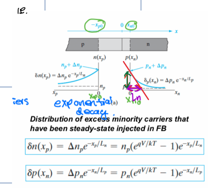
$$P(x) = P_{n0}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{-\frac{x_n}{L_p}} + P_{n0}$$

$$\frac{dP(x)}{dx_n} = -\frac{1}{L_p}P_{n0}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{-\frac{x_n}{L_p}}$$

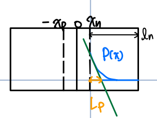
$$\left.\frac{dP(x)}{dx_n}\right|_{x_n=0} = -\frac{1}{L_p}P_{n0}\left(e^{\frac{qV_a}{kT}} - 1\right)$$

means →

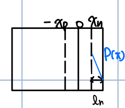
ie.



For a short diode



if $l_n \ll L_p$ ⇒



So, for regular PN Diode : $J_D = q n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_N}{L_p N_A}\right)\left(e^{\frac{V_a}{V_T}} - 1\right)$

for regular Short Diode : $J_D = q n_i^2 \left(\frac{D_p}{l_n N_D} + \frac{D_N}{l_p N_A}\right)\left(e^{\frac{V_a}{V_T}} - 1\right)$